2019
7/8 MATH

SUMMER
ENRICHMENT
PACKET
(OPTIONAL)

MATHCOUNTS TOOLBOX

Facts, Formulas and Tricks

FORMS OF ANSWERS

The following rules explain acceptable forms for answers. Coaches should ensure that Mathletes are familiar with these rules prior to participating at any level of competition. Judges will score competition answers in compliance with these rules for forms of answers.

All answers must be expressed in simplest form. A "common fraction" is to be considered a fraction in the form $\pm\frac{a}{b}$, where a and b are natural numbers and GCF(a, b) = 1. In some cases the term "common fraction" is to be considered a fraction in the form $\frac{A}{B}$, where A and B are algebraic expressions and A and B do not have a common factor. A simplified "mixed number" ("mixed numeral," "mixed fraction") is to be considered a fraction in the form $\pm N\frac{a}{b}$, where N, a and b are natural numbers, $a < b$ and GCF(a, b) = 1. Examples:

Problem: Express 8 divided by 12 as a common fraction. *Answer:* $\frac{2}{3}$ *Unacceptable:* $\frac{4}{6}$

Problem: Express 12 divided by 8 as a common fraction. *Answer:* $\frac{3}{2}$ *Unacceptable:* $\frac{12}{8}$, $1\frac{1}{2}$

Problem: Express the sum of the lengths of the radius and the circumference of a circle with a diameter of $\frac{1}{4}$ as a common fraction in terms of π. *Answer:* $\frac{1+2\pi}{8}$

Problem: Express 20 divided by 12 as a mixed number. *Answer:* $1\frac{2}{3}$ *Unacceptable:* $1\frac{8}{12}$, $\frac{5}{3}$

Ratios should be expressed as simplified common fractions unless otherwise specified. Examples:

Simplified, Acceptable Forms: $\frac{7}{2}$, $\frac{3}{\pi}$, $\frac{4-\pi}{6}$ *Unacceptable:* $3\frac{1}{2}$, $\frac{\frac{1}{4}}{3}$, 3.5, 2:1

Radicals must be simplified. A simplified radical must satisfy: 1) no radicands have a factor which possesses the root indicated by the index; 2) no radicands contain fractions; and 3) no radicals appear in the denominator of a fraction. Numbers with fractional exponents are *not* in radical form. Examples:
Problem: Evaluate $\sqrt{15} \times \sqrt{5}$. *Answer:* $5\sqrt{3}$ *Unacceptable:* $\sqrt{75}$

Answers to problems asking for a response in the form of a dollar amount or an unspecified monetary unit (e.g., "How many dollars...," "How much will it cost...," "What is the amount of interest...") should be expressed in the form ($) *a.bc*, where *a* is an integer and *b* and *c* are digits. The *only* exceptions to this rule are when a is zero, in which case it may be omitted, or when b and c are both zero, in which case they may both be omitted. Examples:
Acceptable: 2.35, 0.38, .38, 5.00, 5 *Unacceptable:* 4.9, 8.0

Units of measurement are not required in answers, but they must be correct if given. When a problem asks for an answer expressed in a specific unit of measure or when a unit of measure is provided in the answer blank, equivalent answers expressed in other units are not acceptable. For example, if a problem asks for the number of ounces and 36 oz is the correct answer, 2 lb 4 oz will not be accepted. If a problem asks for the number of cents and 25 cents is the correct answer, $0.25 will not be accepted.

Do not make approximations for numbers (e.g., π, $\frac{2}{3}$, $5\sqrt{3}$) in the data given or in solutions unless the problem says to do so.

Do not do any intermediate rounding (other than the "rounding" a calculator performs) when calculating solutions. All rounding should be done at the end of the calculation process.

Scientific notation should be expressed in the form $a \times 10^n$ where a is a decimal, $1 \le |a| < 10$, and n is an integer. Examples:
Problem: Write 6895 in scientific notation. *Answer:* 6.895×10^3
Problem: Write 40,000 in scientific notation. *Answer:* 4×10^4 *or* 4.0×10^4

An answer expressed to a greater or lesser degree of accuracy than called for in the problem will not be accepted. Whole-number answers should be expressed in their whole-number form.
Thus, 25.0 will not be accepted for 25, and 25 will not be accepted for 25.0.

The plural form of the units will always be provided in the answer blank, even if the answer appears to require the singular form of the units.

VOCABULARY AND FORMULAS

The following list is representative of terminology used in the problems but should not be viewed as all-inclusive. It is recommended that coaches review this list with their Mathletes.

absolute value
acute angle
additive inverse (opposite)
adjacent angles
algorithm
alternate exterior angles
alternate interior angles
altitude (height)
apex
area
arithmetic mean
arithmetic sequence
base 10
binary
bisect
box-and-whisker plot
center
chord
circle
circumference
circumscribe
coefficient
collinear
combination
common denominator
common divisor
common factor
common fraction
common multiple
complementary angles
composite number
compound interest
concentric
cone
congruent
convex
coordinate plane/system
coordinates of a point
coplanar
corresponding angles
counting numbers
counting principle
cube
cylinder
decagon
decimal
degree measure
denominator
diagonal of a polygon
diagonal of a polyhedron
diameter
difference
digit
digit-sum
direct variation
dividend
divisible
divisor
dodecagon
dodecahedron
domain of a function
edge
endpoint
equation
equiangular
equidistant
equilateral
evaluate
expected value
exponent
expression
exterior angle of a polygon
factor
factorial
finite
formula
frequency distribution
frustum
function
GCF
geometric mean
geometric sequence
height (altitude)
hemisphere
heptagon
hexagon
hypotenuse
image(s) of a point (points) (under a transformation)
improper fraction
inequality
infinite series
inscribe
integer
interior angle of a polygon
interquartile range
intersection
inverse variation
irrational number
isosceles
lateral edge
lateral surface area
lattice point(s)
LCM
linear equation
mean
median of a set of data
median of a triangle
midpoint
mixed number
mode(s) of a set of data
multiple
multiplicative inverse (reciprocal)
natural number
nonagon
numerator
obtuse angle
octagon
octahedron
odds (probability)
opposite of a number (additive inverse)
ordered pair
origin
palindrome
parallel
parallelogram
Pascal's triangle
pentagon
percent increase/decrease
perimeter
permutation
perpendicular
planar
polygon
polyhedron
prime factorization
prime number

principal square root
prism
probability
product
proper divisor
proper factor
proper fraction
proportion
pyramid
Pythagorean Triple
quadrant
quadrilateral
quotient
radius
random
range of a data set
range of a function
rate
ratio
rational number
ray
real number
reciprocal (multiplicative inverse)
rectangle
reflection
regular polygon
relatively prime
remainder
repeating decimal
revolution
rhombus
right angle
right circular cone
right circular cylinder
right polyhedron
right triangle
rotation
scalene triangle
scientific notation
sector
segment of a circle
segment of a line
semicircle
sequence
set
significant digits
similar figures
simple interest
slope
slope-intercept form
solution set
sphere
square
square root
stem-and-leaf plot
sum
supplementary angles
system of equations/inequalities
tangent figures
tangent line
term
terminating decimal
tetrahedron
total surface area
transformation
translation
trapezoid
triangle
triangular numbers
trisect
twin primes
union
unit fraction
variable
vertex
vertical angles
volume
whole number
x-axis
x-coordinate
x-intercept
y-axis
y-coordinate
y-intercept

The list of formulas below is representative of those needed to solve MATHCOUNTS problems but should not be viewed as the only formulas that may be used. Many other formulas that are useful in problem solving should be discovered and derived by Mathletes.

CIRCUMFERENCE

Circle	$C = 2 \times \pi \times r = \pi \times d$

AREA

Square	$A = s^2$
Rectangle	$A = l \times w = b \times h$
Parallelogram	$A = b \times h$
Trapezoid	$A = \frac{1}{2}(b_1 + b_2) \times h$
Circle	$A = \pi \times r^2$
Triangle	$A = \frac{1}{2} \times b \times h$
Triangle	$A = \sqrt{s(s-a)(s-b)(s-c)}$
Equilateral triangle	$A = \dfrac{s^2\sqrt{3}}{4}$
Rhombus	$A = \frac{1}{2} \times d_1 \times d_2$

SURFACE AREA AND VOLUME

Sphere	$SA = 4 \times \pi \times r^2$
Sphere	$V = \frac{4}{3} \times \pi \times r^3$
Rectangular prism	$V = l \times w \times h$
Circular cylinder	$V = \pi \times r^2 \times h$
Circular cone	$V = \frac{1}{3} \times \pi \times r^2 \times h$
Pyramid	$V = \frac{1}{3} \times B \times h$

Pythagorean Theorem	$c^2 = a^2 + b^2$
Counting/ Combinations	${}_nC_r = \dfrac{n!}{r!(n-r)!}$

I. PRIME NUMBERS from 1 through 100 (**1 is not prime!**)

2	3	5	7
11	13	17	19
	23	29	
	31	37	
41	43	47	
	53	59	
	61	67	
71	73	79	
	83	89	
	97		

II.

FRACTIONS	DECIMALS	PERCENTS
$^1/_2$	.5	50 %
$^1/_3$	$.\overline{3}$	$33.\overline{3}$ %
$^2/_3$	$.\overline{6}$	$66.\overline{6}$ %
$^1/_4$	.25	25 %
$^3/_4$	.75	75 %
$^1/_5$	.2	20 %
$^2/_5$	.4	40 %
$^3/_5$	.6	60 %
$^4/_5$	.8	80 %
$^1/_6$	$.1\overline{6}$	$16.\overline{6}$ %
$^5/_6$	$.8\overline{3}$	$83.\overline{3}$ %
$^1/_8$	.125	12.5 %
$^3/_8$	.375	37.5 %
$^5/_8$	.625	62.5 %
$^7/_8$	.875	87.5 %
$^1/_9$	$.\overline{1}$	$11.\overline{1}$ %
$^1/_{10}$	.1	10 %
$^1/_{11}$	$.\overline{09}$	$9.\overline{09}$ %
$^1/_{12}$	$.08\overline{3}$	$8.\overline{3}$ %
$^1/_{16}$	.0625	6.25 %
$^1/_{20}$	.05	5 %
$^1/_{25}$	.04	4 %
$^1/_{50}$	.02	2 %

III. PERFECT SQUARES AND PERFECT CUBES

$1^2 = 1$	$2^2 = 4$	$3^2 = 9$	$4^2 = 16$	$5^2 = 25$
$6^2 = 36$	$7^2 = 49$	$8^2 = 64$	$9^2 = 81$	$10^2 = 100$
$11^2 = 121$	$12^2 = 144$	$13^2 = 169$	$14^2 = 196$	$15^2 = 225$
$16^2 = 256$	$17^2 = 289$	$18^2 = 324$	$19^2 = 361$	$20^2 = 400$
$21^2 = 441$	$22^2 = 484$	$23^2 = 529$	$24^2 = 576$	$25^2 = 625$

$1^3 = 1$	$2^3 = 8$	$3^3 = 27$	$4^3 = 64$	$5^3 = 125$
$6^3 = 216$	$7^3 = 343$	$8^3 = 512$	$9^3 = 729$	$10^3 = 1000$

IV. SQUARE ROOTS

$\sqrt{1} = 1$	$\sqrt{2} \approx 1.414$	$\sqrt{3} \approx 1.732$	$\sqrt{4} = 2$	$\sqrt{5} \approx 2.236$
$\sqrt{6} \approx 2.449$	$\sqrt{7} \approx 2.646$	$\sqrt{8} \approx 2.828$	$\sqrt{9} = 3$	$\sqrt{10} \approx 3.162$

V. FORMULAS

Perimeter:		Volume:	
Triangle	$p = a + b + c$	Cube	$V = s^3$
Square	$p = 4s$	Rectangular Prism	$V = lwh$
Rectangle	$p = 2l + 2w$	Cylinder	$V = \pi r^2 h$
Circle *(circumference)*	$c = 2\pi r$	Cone	$V = (^1/_3)\pi r^2 h$
	$c = \pi d$	Sphere	$V = (^4/_3)\pi r^3$
		Pyramid	$V = (^1/_3)(\text{area of base})h$

Area:			
Rhombus	$A = (½)d_1 d_2$	Circle	$A = \pi r^2$
Square	$A = s^2$	Triangle	$A = (½)bh$
Rectangle	$A = lw = bh$	Right Triangle	$A = (½)l_1 l_2$
Parallelogram	$A = bh$	Equilateral Triangle	$A = (¼)\, s^2 \sqrt{3}$
Trapezoid	$A = (½)(b_1 + b_2)h$		

Total Surface Area:		Lateral Surface Area:	
Cube	$T = 6s^2$	Rectangular Prism	$L = (2l + 2w)h$
Rectangular Prism	$T = 2lw + 2lh + 2wh$	Cylinder	$L = 2\pi rh$
Cylinder	$T = 2\pi r^2 + 2\pi rh$		
Sphere	$T = 4\pi r^2$		

Distance = Rate × Time

Slope of a Line with Endpoints (x_1, y_1) and (x_2, y_2): slope $= m = \dfrac{(y_2 - y_1)}{(x_2 - x_1)}$

Distance Formula: distance between two points or length of segment with endpoints (x_1, y_1) and (x_2, y_2)

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Midpoint Formula: midpoint of a line segment given two endpoints (x_1, y_1) and (x_2, y_2)

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Circles:

Length of an arc $= \left(\dfrac{x}{360}\right)(2\pi r)$, where x is the measure of the central angle of the arc

Area of a sector $= \left(\dfrac{x}{360}\right)(\pi r^2)$, where x is the measure of the central angle of the sector

Combinations (number of groupings when the order of the items in the groups does not matter):

Number of combinations = $\dfrac{N!}{R!(N-R)!}$, where N = # of total items and R = # of items being chosen

Permutations (number of groupings when the order of the items in the groups matters):

Number of permutations = $\dfrac{N!}{(N-R)!}$, where N = # of total items and R = # of items being chosen

Length of a Diagonal of a Square = $s\sqrt{2}$

Length of a Diagonal of a Cube = $s\sqrt{3}$

Length of a Diagonal of a Rectangular Solid = $\sqrt{x^2+y^2+z^2}$, with dimensions x, y and z

Number of Diagonals for a Convex Polygon with N Sides = $\dfrac{N(N-3)}{2}$

Sum of the Measures of the Interior Angles of a Regular Polygon with N Sides = $(N-2)180$

Heron's Formula:

For **any triangle** with side lengths a, b and c, $Area = \sqrt{s(s-a)(s-b)(s-c)}$, where $s = ½(a+b+c)$

Pythagorean Theorem: (Can be used with **all right triangles**)

$a^2+b^2=c^2$, where a and b are the lengths of the legs and c is the length of the hypotenuse

Pythagorean Triples: Integer-length sides for right triangles form Pythagorean Triples – the largest number must be on the hypotenuse. Memorizing the bold triples will also lead to other triples that are multiples of the original.

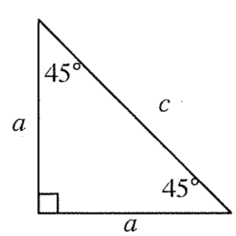
3	**4**	**5**	**5**	**12**	**13**	**7**	**24**	**25**
6	8	10	10	24	26	**8**	**15**	**17**
9	12	15	15	36	39	**9**	**40**	**41**

Special Right Triangles:

45° – 45° – 90°

hypotenuse = $\sqrt{2}$ (leg) = $a\sqrt{2}$

leg = $\dfrac{\text{hypotenuse}}{\sqrt{2}} = \dfrac{c}{\sqrt{2}}$

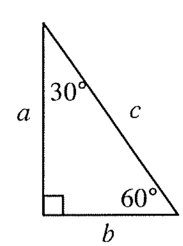


30° – 60° – 90°

hypotenuse = 2(shorter leg) = $2b$

longer leg = $\sqrt{3}$ (shorter leg) = $b\sqrt{3}$

shorter leg = $\dfrac{\text{longer leg}}{\sqrt{3}} = \dfrac{\text{hypotenuse}}{2}$



Geometric Mean: $\frac{a}{x} = \frac{x}{b}$ therefore, $x^2 = ab$ and $x = \sqrt{ab}$

Regular Polygon: Measure of a central angle $= \frac{360}{n}$, where n = number of sides of the polygon

Measure of vertex angle $= 180 - \frac{360}{n}$, where n = number of sides of the polygon

Ratio of Two Similar Figures: If the ratio of the measures of corresponding side lengths is $A{:}B$, then the ratio of the perimeters is $A{:}B$, the ratio of the areas is $A^2 : B^2$ and the ratio of the volumes is $A^3 : B^3$.

Difference of Two Squares: $a^2 - b^2 = (a-b)(a+b)$

Example: $12^2 - 9^2 = (12-9)(12+9) = 3 \cdot 21 = 63$

$144 - 81 = 63$

Determining the Greatest Common Factor (GCF): 5 Methods

1. Prime Factorization (Factor Tree) – Collect all common factors
2. Listing all Factors
3. Multiply the two numbers and divide by the Least Common Multiple (LCM)
   Example: to find the GCF of 15 and 20, multiply $15 \times 20 = 300$, then divide by the LCM, 60. The GCF is 5.
4. Divide the smaller number into the larger number. If there is a remainder, divide the remainder into the divisor until there is no remainder left. The last divisor used is the GCF.

   Example:

$180\overline{)385}$	$25\overline{)180}$	$5\overline{)25}$
360	175	25
25	5	0

   5 is the GCF of 180 and 385

5. Single Method for finding both the GCF and LCM
   Put both numbers in a lattice. On the left, put ANY divisor of the two numbers and put the quotients below the original numbers. Repeat until the quotients have no common factors except 1 (relatively prime). Draw a "boot" around the left-most column and the bottom row. Multiply the vertical divisors to get the GCF. Multiply the "boot" numbers (vertical divisors and last-row quotients) to get the LCM.

	40	**140**
2	20	70

	40	**140**
2	20	70
10		

	40	**140**
2	20	70
10	2	7

The GCF is $2 \times 10 = 20$
The LCM is $2 \times 10 \times 2 \times 7 = 280$

VI. DEFINITIONS

Real Numbers: all rational and irrational numbers

Rational Numbers: numbers that can be written as a ratio of two integers

Irrational Numbers: non-repeating, non-terminating decimals; can't be written as a ratio of two integers (i.e. $\sqrt{7}$, π)

Integers: {..., -3, -2, -1, 0, 1, 2, 3, ...}

Whole Numbers: {0, 1, 2, 3, ...}

Natural Numbers: {1, 2, 3, 4, ...}

Common Fraction: a fraction in lowest terms (Refer to "Forms of Answers" in the *MATHCOUNTS School Handbook* for a complete definition.)

Equation of a Line:

Standard form: $Ax + By = C$ with slope = $-\frac{A}{B}$

Slope-intercept form: $y = mx + b$ with slope = m and y-intercept = b

Regular Polygon: a convex polygon with all equal sides and all equal angles

Negative Exponents: $x^{-n} = \frac{1}{x^n}$ *and* $\frac{1}{x^{-n}} = x^n$

Systems of Equations:

$$\begin{array}{rl} x + y &= 10 \\ x - y &= 6 \\ \hline 2x &= 16 \\ \mathbf{x} &= \mathbf{8} \end{array} \quad \longrightarrow \quad \begin{array}{rl} 8 + y &= 10 \\ \mathbf{y} &= \mathbf{2} \end{array}$$

(8, 2) is the solution of the system

Mean = Arithmetic Mean = Average

Mode = the number(s) occurring the most often; there may be more than one

Median = the middle number when written from least to greatest
If there is an even number of terms, the median is the average of the two middle terms.

Range = the difference between the greatest and least values

Measurements:

1 mile = 5280 feet
1 square foot = 144 square inches
1 square yard = 9 square feet
1 cubic yard = 27 cubic feet

VII. PATTERNS

Divisibility Rules:

Number is divisible by 2: last digit is 0,2,4,6 or 8
3: sum of digits is divisible by 3
4: two-digit number formed by the last two digits is divisible by 4
5: last digit is 0 or 5
6: number is divisible by **both** 2 and 3
8: three-digit number formed by the last 3 digits is divisible by 8
9: sum of digits is divisible by 9
10: last digit is 0

Sum of the First N Odd Natural Numbers = N^2

Sum of the First N Even Natural Numbers = $N^2 + N = N(N + 1)$

Sum of an Arithmetic Sequence of Integers: $\frac{N}{2} \times$ (first term + last term), where N = amount of numbers/terms in the sequence

Find the digit in the units place of a particular power of a particular integer

Find the pattern of units digits:
7^1 ends in 7
7^2 ends in 9
7^3 ends in 3
7^4 ends in 1
7^5 ends in 7

(pattern repeats every 4 exponents)

Divide 4 into the given exponent and compare the remainder with the first four exponents. (a remainder of 0 matches with the exponent of 4)

Example: What is the units digit of 7^{22}?

22 $\div$ 4 = 5 r. **2**, so the units digit of 7^{22} is the same as the units digit of 7^2, which is 9.

VIII. FACTORIALS ("$n!$" is read "n factorial")

$n! = (n)\times(n-1)\times(n-2)\times\ldots\times(2)\times(1)$ Example: $5! = 5\times4\times3\times2\times1 = 120$

$0! = 1$
$1! = 1$
$2! = 2$
$3! = 6$
$4! = 24$
$5! = 120$
$6! = 720$
$7! = 5040$

Notice $\frac{6!}{4!} = \frac{6\times5\times\cancel{4\times3\times2\times1}}{\cancel{4\times3\times2\times1}} = 30$

IX. PASCAL'S TRIANGLE

Pascal's Triangle Used for Probability:

Remember that the first row is row zero (0). Row 4 is 1 4 6 4 1. This can be used to determine the different outcomes when flipping four coins.

1	**4**	**6**	**4**	**1**
way to get 4 heads 0 tails	ways to get 3 heads 1 tail	ways to get 2 heads 2 tails	ways to get 1 head 3 tails	way to get 0 heads 4 tails

For the Expansion of $(a + b)^n$, use numbers in Pascal's Triangle as coefficients.

1	$(a+b)^0 = 1$
1 1	$(a+b)^1 = a + b$
1 2 1	$(a+b)^2 = a^2 + 2ab + b^2$
1 3 3 1	$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$
1 4 6 4 1	$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$
1 5 10 10 5 1	$(a+b)^5 = a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$

For 2^n, add all the numbers in the n^{th} row. (Remember the triangle starts with row 0.)

1	$2^0 = 1$
1 1	$2^1 = 1 + 1 = 2$
1 2 1	$2^2 = 1 + 2 + 1 = 4$
1 3 3 1	$2^3 = 1 + 3 + 3 + 1 = 8$
1 4 6 4 1	$2^4 = 1 + 4 + 6 + 4 + 1 = 16$
1 5 10 10 5 1	$2^5 = 1 + 5 + 10 + 10 + 5 + 1 = 32$

X. SQUARING A NUMBER WITH A UNITS DIGIT OF 5

$(n5)^2 = \underline{n \times (n + 1)}\ \underline{2}\ \underline{5}$, where n represents the block of digits before the units digit of 5

Examples:

$(35)^2 = \underline{3\times(3+1)}\ \underline{2}\ \underline{5}$
$= \underline{3\times(4)}\ \underline{2}\ \underline{5}$
$= \underline{1\,2}\ \underline{2}\ \underline{5}$
$= 1{,}225$

$(125)^2 = \underline{12\times(12+1)}\ \underline{2}\ \underline{5}$
$= \underline{12\times(13)}\ \underline{2}\ \underline{5}$
$= \underline{1\,5\,6}\ \underline{2}\ \underline{5}$
$= 15{,}625$

XI. BASES

Base 10 = decimal – only uses digits 0 – 9

Base 2 = binary – only uses digits 0 – 1

Base 8 = octal – only uses digits 0 – 7

Base 16 = hexadecimal – only uses digits 0 – 9, A – F (where A=10, B=11, …, F=15)

Changing from Base 10 to Another Base:

What is the base 2 representation of 125 (or "125 base 10" or "125_{10}")?

We know $125 = 1(10^2) + 2(10^1) + 5(10^0) = 100 + 20 + 5$, but what is it equal to in base 2?

$125_{10} = ?(2^n) + ?(2^{n-1}) + \ldots + ?(2^0)$

The largest power of 2 in 125 is $64 = 2^6$, so we now know our base 2 number will be:

$?(2^6) + ?(2^5) + ?(2^4) + ?(2^3) + ?(2^2) + ?(2^1) + ?(2^0)$ and it will have 7 digits of 1's and/or 0's.

Since there is **one** 64, we have: $\mathbf{1}(2^6) + ?(2^5) + ?(2^4) + ?(2^3) + ?(2^2) + ?(2^1) + ?(2^0)$

We now have 125 – 64 = 61 left over, which is **one** $32 = 2^5$ and 29 left over, so we have:

$\mathbf{1}(2^6) + \mathbf{1}(2^5) + ?(2^4) + ?(2^3) + ?(2^2) + ?(2^1) + ?(2^0)$

In the left-over 29, there is **one** $16 = 2^4$, with 13 left over, so we have:

$\mathbf{1}(2^6) + \mathbf{1}(2^5) + \mathbf{1}(2^4) + ?(2^3) + ?(2^2) + ?(2^1) + ?(2^0)$

In the left-over 13, there is **one** $8 = 2^3$, with 5 left over, so we have:

$\mathbf{1}(2^6) + \mathbf{1}(2^5) + \mathbf{1}(2^4) + \mathbf{1}(2^3) + ?(2^2) + ?(2^1) + ?(2^0)$

In the left-over 5, there is **one** $4 = 2^2$, with 1 left over, so we have:

$\mathbf{1}(2^6) + \mathbf{1}(2^5) + \mathbf{1}(2^4) + \mathbf{1}(2^3) + \mathbf{1}(2^2) + ?(2^1) + ?(2^0)$

In the left-over 1, there is **no** $2 = 2^1$, so we still have 1 left over, and our expression is:

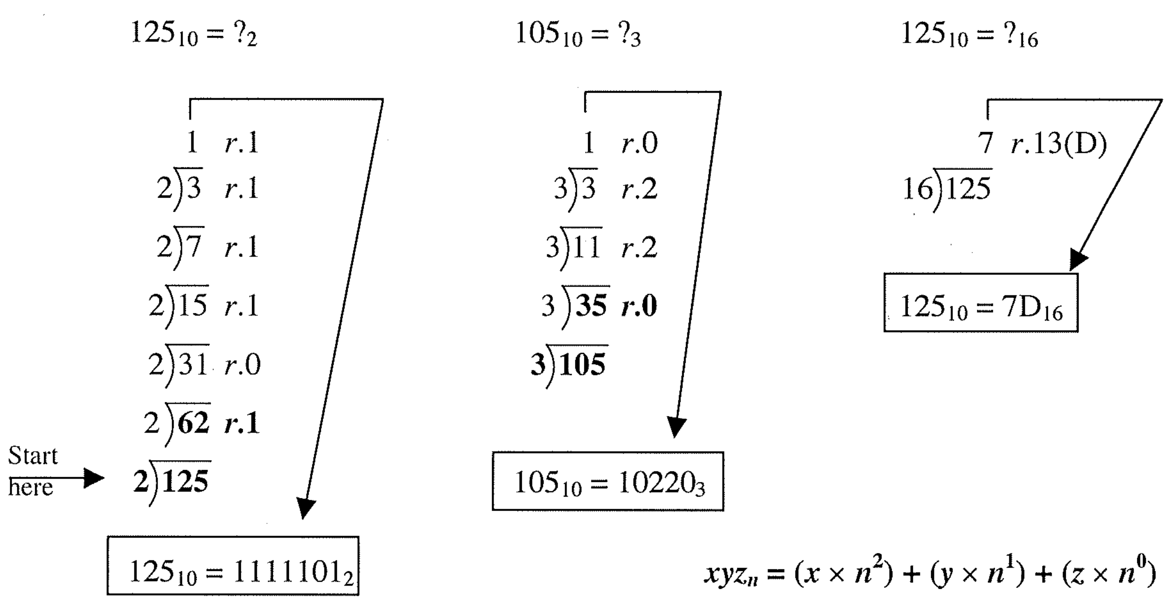
$\mathbf{1}(2^6) + \mathbf{1}(2^5) + \mathbf{1}(2^4) + \mathbf{1}(2^3) + \mathbf{1}(2^2) + \mathbf{0}(2^1) + ?(2^0)$

The left-over 1 is **one** 2^0, so we finally have:

$\mathbf{1}(2^6) + \mathbf{1}(2^5) + \mathbf{1}(2^4) + \mathbf{1}(2^3) + \mathbf{1}(2^2) + \mathbf{0}(2^1) + \mathbf{1}(2^0) = 1111101_2$

Now try ***What is the base 3 representation of 105?***
The largest power of 3 in 105 is $81 = 3^4$, so we now know our base 3 number will be:
$?(3^4) + ?(3^3) + ?(3^2) + ?(3^1) + ?(3^0)$ and will have 5 digits of 2's, 1's, and/or 0's.
Since there is **one** 81, we have: $\mathbf{1}(3^4) + ?(3^3) + ?(3^2) + ?(3^1) + ?(3^0)$
In the left-over $105 - 81 = 24$, there is **no** $27 = 3^3$, so we still have 24 and the expression:
$\mathbf{1}(3^4) + \mathbf{0}(3^3) + ?(3^2) + ?(3^1) + ?(3^0)$
In the left-over 24, there are **two** 9's (or 3^2's), with 6 left over, so we have:
$\mathbf{1}(3^4) + \mathbf{0}(3^3) + \mathbf{2}(3^2) + ?(3^1) + ?(3^0)$
In the left-over 6, there are **two** 3's (or 3^1's), with 0 left over, so we have:
$\mathbf{1}(3^4) + \mathbf{0}(3^3) + \mathbf{2}(3^2) + \mathbf{2}(3^1) + ?(3^0)$
Since there is nothing left over, we have **no** 1's (or 3^0's), so our final expression is:
$\mathbf{1}(3^4) + \mathbf{0}(3^3) + \mathbf{2}(3^2) + \mathbf{2}(3^1) + \mathbf{0}(3^0) = 10220_3$

The following is another fun algorithm for converting base 10 numbers to other bases:



$$xyz_n = (x \times n^2) + (y \times n^1) + (z \times n^0)$$

Notice: Everything in bold shows the first division operation. The first remainder will be the last digit in the base *n* representation, and the quotient is then divided again by the desired base. The process is repeated until a quotient is reached that is less than the desired base. At that time, the final quotient and remainders are read downward.

XII. FACTORS

Determining the Number of Factors of a Number: First find the prime factorization (include the 1 if a factor is to the first power). *Increase* each exponent by 1 and multiply these new numbers together.

Example: How many factors does 300 have?
The prime factorization of 300 is $2^2 \times 3^1 \times 5^2$. Increase each of the exponents by 1 and multiply these new values: $(2+1) \times (1+1) \times (2+1) = 3 \times 2 \times 3 = 18$. So 300 has 18 factors.

Finding the Sum of the Factors of a Number:

Example: What is the sum of the factors of 10,500?
(From the prime factorization $2^2 \times 3^1 \times 5^3 \times 7^1$, we know 10,500 has $3 \times 2 \times 4 \times 2 = 48$ factors.)
The sum of these 48 factors can be calculated from the prime factorization, too:
$(2^0 + 2^1 + 2^2)(3^0 + 3^1)(5^0 + 5^1 + 5^2 + 5^3)(7^0 + 7^1) = 7 \times 4 \times 156 \times 8 = 34{,}944$.

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MATHCOUNTS Foundation
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Alexandria, VA 22314
t 703-684-2828
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e info@mathcounts.org

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NASSP 2003-2004

Competition Components

MATHCOUNTS competitions are designed to be completed in approximately three hours:

The **Sprint Round** (40 minutes) consists of 30 problems. This round tests accuracy, with time being such that only the most capable students will complete all of the problems. **Calculators are not permitted.**

The **Target Round** (approximately 30 minutes) consists of eight problems presented to competitors in four pairs (6 minutes per pair). This round features multi-step problems that engage Mathletes in mathematical reasoning and problem-solving processes. **Problems assume the use of calculators.**

The **Team Round** (20 minutes) consists of 10 problems that team members work together to solve. Team member interaction is permitted and encouraged. **Problems assume the use of calculators.** Note: Coordinators may opt to allow those competing as "individuals" to create a "squad" of four to take the Team Round for the experience, but the round *should not be scored and is not considered official*.

The **Countdown Round** is a fast-paced, oral competition for top-scoring individuals (based on scores in the Sprint and Target Rounds). In this round, pairs of Mathletes compete against each other and the clock to solve problems. **Calculators are not permitted.**

At Chapter and State competitions, a Countdown Round may be conducted officially, unofficially (for fun) or omitted. However, the use of an official Countdown Round will be consistent for all chapters within a state. In other words, *all* chapters within a state must use the round officially in order for *any* chapter within a state to use it officially. All students, whether registered as part of a school team or as an individual competitor, are eligible to qualify for the Countdown Round.

An *official* Countdown Round is defined as one that determines an individual's final overall rank in the competition. If the Countdown Round is used officially, the official procedures as established by the MATHCOUNTS Foundation must be followed.

If a Countdown Round is conducted *unofficially*, the official procedures do not have to be followed. Chapters and states choosing not to conduct the round officially must determine individual winners on the sole basis of students' scores in the Sprint and Target Rounds of the competition.

In an official Countdown Round, the top 25% of students, up to a maximum of 10, are selected to compete. These students are chosen based on their individual scores. The two lowest-ranked students are paired, a question is projected and students are given 45 seconds to solve the problem. A student may buzz in at any time, and if s/he answers correctly, a point is scored; if a student answers incorrectly, the other student has the remainder of the 45 seconds to answer. Three questions are read to each pair of students, one question at a time, and the student who scores the most points (not necessarily 2 out of 3) captures the place, progresses to the next round and challenges the next highest-ranked student. (If students are tied after three questions (at 1-1 or 0-0), questions continue to be read until one is successfully answered.) This procedure continues until the fourth-ranked Mathlete and her/his opponent compete. For the final four rounds, the first student to correctly answer three questions advances. The Countdown Round proceeds until a first-place individual is identified. (More detailed rules regarding the Countdown Round procedure are identified in the "Instructions" section of the School Competition booklet.)

Note: Rules for the Countdown Round change for the National Competition.

The Masters Round is a special round for top individual scorers at the state and national levels. In this round, top individual scorers prepare an oral presentation on a specific topic to be presented to a panel of judges. The Masters Round is optional at the state level; if held, the state coordinator determines the number of Mathletes that participate. At the national level, four Mathletes participate. (Participation in the Masters Round is optional. A student declining to participate will not be penalized.)

Each student is given 30 minutes to prepare his/her presentation. **Calculators may be used.** The presentation will be 15 minutes—up to 11 minutes may be used for the student's oral response to the problem, and the remaining time may be used for questions by the judges. This competition values creativity and oral expression as well as mathematical accuracy. Judging of presentations is based on knowledge, presentation and the response to judges' questions.

Print This Page

You are here: Home > Coaches > Getting Started > Competition Rules & Procedures > Competition Components

Revised: September 26, 2003

MATHCOUNTS®

2012
■ School Competition ■
Sprint Round
Problems 1–30

Name ______________________________

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This section of the competition consists of 30 problems. You will have 40 minutes to complete all the problems. You are not allowed to use calculators, books or other aids during this round. Calculations may be done on scratch paper. All answers must be complete, legible and simplified to lowest terms. Record only final answers in the blanks in the left-hand column of the competition booklet. If you complete the problems before time is called, use the remaining time to check your answers.

In each written round of the competition, the required unit for the answer is included in the answer blank. The plural form of the unit is always used, even if the answer appears to require the singular form of the unit. The unit provided in the answer blank is the only form of the answer that will be accepted.

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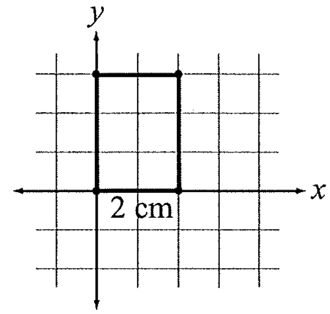
1. ____________ cm²

A rectangle has a perimeter of 10 cm and a width of 2 cm. What is the number of square centimeters in its area?



2. ____________

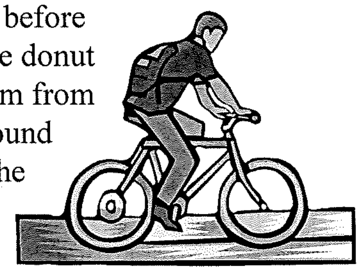
What is the product of all the positive integer factors of 15?

3. ____________

When the sum of $\frac{2}{3}$ and $\frac{5}{24}$ is expressed as a common fraction, what is its denominator?

4. ____________ km

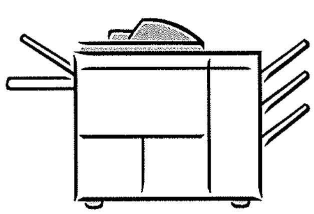
Joaquin rode his bicycle down a straight street to school. He passed the playground, then the library, and then the donut shop before arriving at school. The playground is 0.4 km from the donut shop. The library is 1.0 km from the school and 0.3 km from the donut shop. What is the distance from the playground to the school? Express your answer as a decimal to the nearest tenth.



5. ____________ orders

One recipe for Tex-Mex four layer dip calls for one layer each of refried beans, sour cream, guacamole and tomatoes. If the refried beans must be the bottom layer, in how many different orders can the dip be constructed?

6. ____________ minutes



A Sprint copier can copy 2400 pages in 60 minutes. At this rate, how many minutes will it take for a Sprint copier to copy 120 pages?

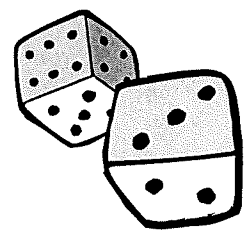
7. ______________ What is the value of $3^4 - 2^4$?

8. ______________ What is the product of the largest and smallest prime factors of 165?

9. ______________ Felix's test scores for the second quarter are 87, 92, 88, 90, 83, 90, 86. What is the mean of the median and the mode of these scores?

10. ___________ pairs How many pairs of distinct integers chosen from the set of odd integers between 6 and 16 have a sum greater than 23?

11. ______________ If you toss two standard six-sided dice, what is the probability that you will get a 3 on at least one die? Express your answer as a common fraction.



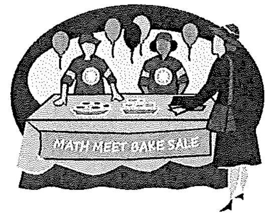
12. ______________ An operation ✪ is defined as a ✪ $b = a + \frac{b}{2}$. What is the value of −8 ✪ 6?

13. ___________ sides If a regular polygon has a total of nine diagonals, how many sides does it have?

14. ______________ The length of a rectangle is three times its width. A new rectangle is created by decreasing the length of the original rectangle by half. By what factor must the original width be multiplied, if the area remains unchanged?

15. ______ brownies

The math team at Mandelbrot Middle School earned \$273.00 by selling a combined total of 440 brownies and cookies during their math meet. If each brownie sold for \$0.75, and each cookie sold for \$0.50, how many brownies did they sell?



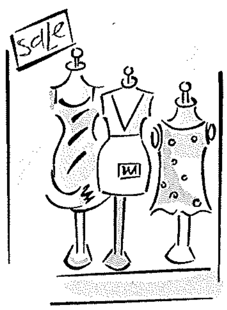
16. (,)

A line with slope $\frac{2}{3}$ passes through the point $(1,2)$. At what point does this line cross the x-axis? Express your answer as an ordered pair.

17. ______

If twice a number is equal to 6 more than half the number, what is the number?

18. ______



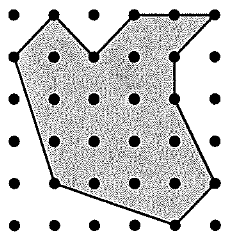
In a boutique, items were put in a showcase, and each was assigned a price for January. Each month after that, the price was 10 percent less than the price for the previous month. What is the ratio of the price of an item in April to the price of the same item in January? Express your answer as a common fraction.

19. ______ in

Giuseppe's dad is 6'0" tall and his mother is 5'4" tall. One formula used to predict a child's adult height is to take the average of the height of the parent of the opposite gender and $\frac{13}{12}$ the height of the parent of the same gender. Using this formula, what is Giuseppe's expected adult height, in inches?

20. ____________ What is the ratio of the number of positive divisors of 24 to the number of positive divisors of 36? Express your answer as a common fraction.

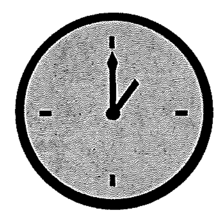
21. ____________ units^2



In the figure shown, the distance between adjacent dots in each row and in each column is 1 unit. In square units, what is the area of the shaded region? Express your answer as a mixed number.

22. ____________ units The numerical value of the area of a square is 36 less than three times the numerical value of its perimeter. What is the perimeter of the square?

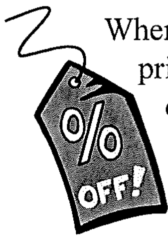
23. ____________ in A clock's minute hand has a length of 2 in. It takes 15 minutes for the minute hand to rotate clockwise 90 degrees. What is the distance traveled by the tip of the minute hand in a 15-minute period? Express your answer in terms of π.



24. ____________ The ratio of John's allowance to Bill's allowance is 3:7. The ratio of John's allowance to Mary's allowance is 2:5. What is the ratio of Mary's allowance to Bill's allowance? Express your answer as a common fraction.

25. ____________ cm³ A cone has a volume of 36π cm^3 and a height of 3 cm. A cylinder has a radius equal to half the radius of the cone, and a height equal to twice the height of the cone. What is the positive difference between the volume of the cone and the volume of the cylinder? Express your answer in terms of π.

26. ____________ %



When a clothing store first makes an item available for purchase, the price is marked up 60% above the cost to the store. What discount can the store offer so that the discounted price is the same as the original cost to the store? Express your answer as a percent to the nearest tenth.

27. ____________ If $8 - x = -2x - 8 + 5x$, what is the value of x?

28. ____________ For what value of n does $\dfrac{10!}{7!3!} = n!$?

29. ____________ The difference of the squares of two distinct positive numbers is equal to twice the square of their difference. What is the ratio of the smaller number to the larger? Express your answer as a common fraction.

30. ____________ units The area of a particular regular hexagon is x^3 square units, where x is the measure of the distance from the center of the hexagon to the midpoint of a side. What is the side length of the hexagon?

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2012
■ School Competition ■
Target Round
Problems 1 and 2

Name ______________________________

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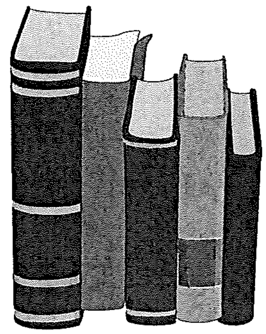
This section of the competition consists of eight problems, which will be presented in pairs. Work on one pair of problems will be completed and answers will be collected before the next pair is distributed. The time limit for each pair of problems is six minutes. The first pair of problems is on the other side of this sheet. When told to do so, turn the page over and begin working. This round assumes the use of calculators, and calculations also may be done on scratch paper, but no other aids are allowed. All answers must be complete, legible and simplified to lowest terms. Record only final answers in the blanks in the left-hand column of the problem sheets. If you complete the problems before time is called, use the time remaining to check your answers.

Total Correct	Scorer's Initials

1. _____________ combinations

Mrs. Libro's students may write book reports on any three books from a list of five books. How many combinations of three books are possible?



2. _____________

Jennifer has taken three exams and earned scores of 92, 77 and 95. She has to take a final exam which will count as two exam scores. What score does she need to earn on the final exam to have an exam average of 90 for the year?

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2012
■ School Competition ■
Target Round
Problems 3 and 4

Name ______________________________

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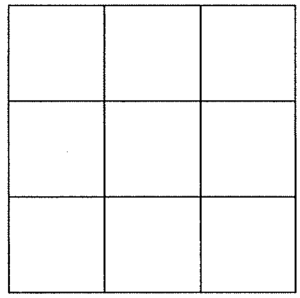
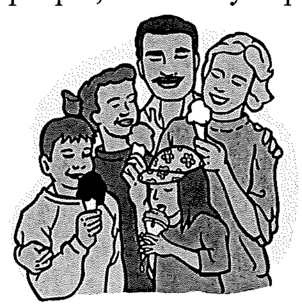
3. ________ rectangles How many rectangles are there in the 3×3 grid made of 9 congruent squares, shown here?



4. ________ cups If $2\frac{1}{2}$ cups of milk are needed to make ice cream for 5 people, how many cups of milk are needed to make ice cream for 20 people?



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Target Round

Problems 5 and 6

Name ______________________________

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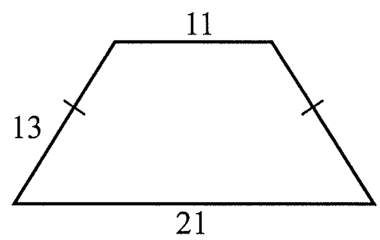
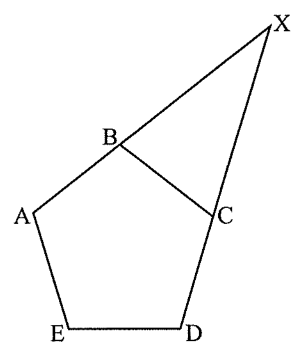
5. ____________ units An isosceles trapezoid has bases of length 11 units and 21 units and legs of length 13 units. In units, what is the height of the trapezoid?



6. ____________ degrees When sides AB and DC of regular pentagon ABCDE are extended, they intersect at point X. What is the degree measure of $\angle X$?



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2012
■ School Competition ■
Target Round
Problems 7 and 8

Name ______________________________

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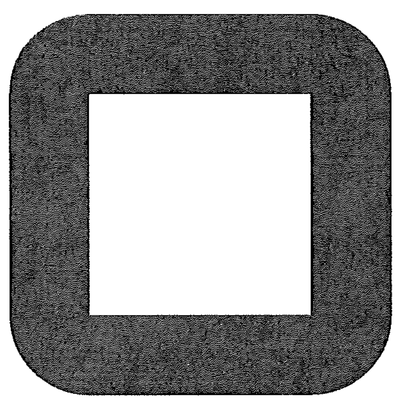
7. ______________ If x is negative and $x^2 = 16$, what is the value of $x^3 + \sqrt{-x}$?

8. ______________ in^2 A 4 in × 4 in square is surrounded by a border consisting of all points in the plane of the square that are within 1.5 inches of the square and not in the square. In square inches, what is the area of the border? Express your answer as a decimal to the nearest tenth.



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2012
■ School Competition ■
Team Round
Problems 1–10

Team Members ______________________________ , Captain

____

____

____

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This section of the competition consists of 10 problems which the team has 20 minutes to complete. Team members may work together in any way to solve the problems. Team members may talk to each other during this section of the competition. This round assumes the use of calculators, and calculations also may be done on scratch paper, but no other aids are allowed. All answers must be complete, legible and simplified to lowest terms. The team captain must record the team's official answers on his/her own competition booklet, which is the only booklet that will be scored. If the team completes the problems before time is called, use the remaining time to check your answers.

Total Correct	Scorer's Initials

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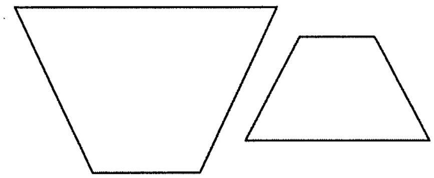
1. ____________ cm

Two trapezoids, shown here, are similar. The ratio of the sum of the bases of the larger trapezoid to the sum of the bases of the smaller trapezoid is 5:4. If the height of the smaller trapezoid is 20 cm, what is the height of the larger trapezoid, in centimeters?



2. ____________

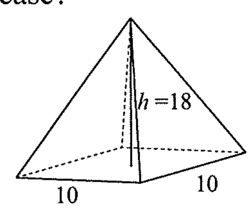
If $3x + 2 = 17$, what is the value of x^3?

3. ____________

The length of segment MC is 6 units with endpoints M(x, 3) and C(−2, 3). What is the value of x if point M is in the first quadrant?

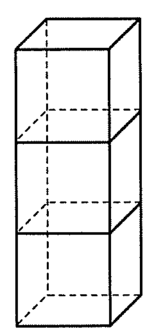
4. ____________ %

If the height of the square pyramid shown is increased by 50% while the length of each side of the base is decreased by 20%, by what percent does the volume of the pyramid decrease?



5. ____________ cm^2

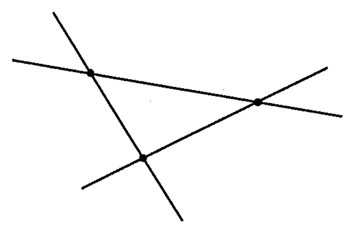
Three cubes, each with a volume of 8 cm^3, are stacked and glued, as shown. What is the total surface area of the resulting prism?



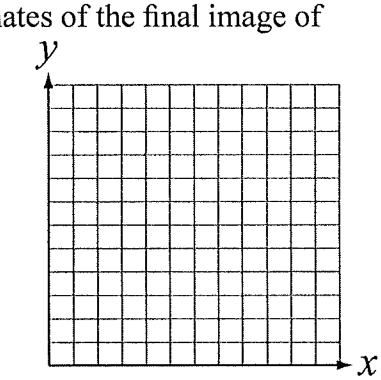
6. ________ pkgs Ray is setting up a hot dog stand for the county fair. If hot dogs come in packages of 10, hot dog buns come in packages of 8, and paper plates come in packages of 36, what is the least positive number of packages of hot dogs that Ray can purchase so there will be the same number of hot dogs, buns and plates?

7. ________ The mean and median of a set of five distinct positive integers is 5. What is the largest integer that can be in the set?

8. ________ points The maximum number of points of intersection determined by three distinct lines in a plane is three. What is the maximum number of points of intersection that are possible with 6 distinct lines?



9. (,) Quadrilateral MNPQ has vertices with coordinates M(2, 5), N(6, 5), P(6, 7) and Q(2, 7). When the figure is rotated clockwise 270° around point M and then reflected across the line $x = 1$, what are the coordinates of the final image of point Q? Express your answer as an ordered pair.



10. ________ What is the largest value of x for which $\sqrt{2x+1} = 2x + 1$?

MATHCOUNTS®

2012
■ Chapter Competition ■
Countdown Round
Problems 1–80

This section contains problems to be used in the Countdown Round.

1. ____________ What is the sum $\frac{2}{7}+\frac{4}{13}+\frac{4}{7}+\frac{2}{13}+\frac{1}{7}+\frac{7}{13}$?

2. ____________ If the four points (2, 7), (7, −3), (1, 9) and (6, y) lie on a line, what is the value of y?

3. ____________ If Louis rolls two standard, six-sided dice once, what is the probability that he will roll a sum of 2, 3 or 12? Express your answer as a common fraction.

4. ____________ What is the value of $5^2 - 5^3$?

5. ________ (dollars) Eighteen people contributed to a certain charity. There were six contributions of \$10, six of \$20 and six of \$30. What is the mean value of the contributions?

6. ________ (mi/h) There are 5280 feet in 1 mile. If Alexis can run at a rate of 528 feet per minute, what is her speed, in miles per hour?

7. ________ (hours) If Josephine makes \$8 an hour at her babysitting job, how many hours will it take her to earn enough money to buy a new xPad that costs \$575? Express your answer to the nearest whole number.

8. ________ (feet) A circular pool is being built inside a rectangular region 12 ft by 18 ft. If the pool is to be as large as possible, and the edge of the pool must be at least 2 ft from each edge of the rectangle, what is the maximum possible value for the radius of the pool, in feet?

9. ________ (bars) A digital clock's display has 23 lighted bars and a colon that can be illuminated to display the time. What is the least number of bars that can be illuminated at any time?

10. ____________ What is the mean of {23, 25, 27, 29, 31}?

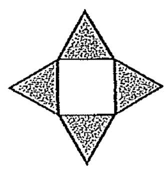
11. ____________ What is the result when 60 is divided by $\frac{1}{2}$ and the result is added to 20?

12. ________ (buckets) A bucket holds 4 quarts of popcorn. If $\frac{1}{3}$ cup of corn kernels makes 2 quarts of popcorn, how many buckets can be filled with the popcorn made from 4 cups of kernels?

13. ______________ What is the sum of the prime factors of 210?

14. ________ (years) Mary is seven years older than her sister. In three years, Mary will be twice as old as her sister will be. In years, how old is Mary now?

15. ________ (lines)



A square is surrounded by four equilateral triangles, as shown. How many lines of symmetry does this figure have?

16. ________ (km/h) Nihal ran 7 kilometers in 21 minutes. What was his average speed, in kilometers per hour?

17. ______________ What is the sum of all positive divisors of 4^3?

18. ________ (sections) A spinner has 15 congruent sections colored either black or red. If the spinner lands on a black section 13 out of 50 spins, what would be the best prediction for the number of sections that are colored red?

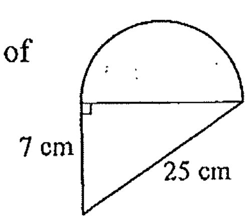
19. ________ (trees) A botanist found that a certain forest contains only pine, spruce, oak and maple trees. These trees appear in a ratio of 3:5:7:5, respectively. Out of 1,000 trees in this forest, how many would be expected to be maple trees?

20. ________ (red marbles) There are 15 marbles in a bag, some green and the rest red. Whenever a pair of marbles is removed from the bag, at least one of the marbles is green. How many red marbles are in the bag?

21. ______________ What is the smallest positive difference between two integers whose product is 2400?

22. ______________ If $a = 12 - 3 \cdot 2$ and $b = 5 + 2^2$, what is $\frac{a^2}{b}$?

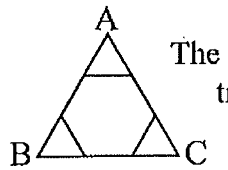
23. ________ (cm^2) A right triangle has a hypotenuse of length 25 cm and one leg of length 7 cm. The other leg of the triangle is the diameter of a semicircle, as shown. What is the area of the semicircle, in square centimeters? Express your answer in terms of π.



24. ____________ Regina told Rinaldo that she was going to show him five numbers, one at a time, and he was to find their product. After seeing the second number, however, Rinaldo already knew the answer. What was the product of the five numbers?

25. ____________ What is $2015^2 - 2013^2$?

26. ________ (cm²)



The diagram shows a regular hexagon inscribed in equilateral triangle ABC. If the area of the hexagon is 60 cm², what is the area of ΔABC, in square centimeters?

27. ________ (points) Marty averaged 85 points on her first three tests. If she averages 90 points on her next two tests, what will her average be for all five tests?

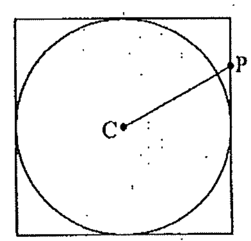
28. ____________ What positive, two-digit integer has exactly 9 distinct factors?

29. ________ (cm) A rectangle has area 48 cm², and its length is three times its width. What is the perimeter of the rectangle, in centimeters?

30. ________ (miles) The ratio of the number of laps on a track to the distance, in miles, an athlete runs is 4:1. How many miles did an athlete run if she ran 22 laps? Express your answer as a decimal to the nearest tenth.

31. ____________ What is the largest square number that is 24 more than another square number?

32. ________ (in) A circle of radius 2 inches has its center at C and is tangent to the sides of a square. A point P is drawn on the square midway between a point of tangency of the circle and one vertex of the square. What is the length of segment CP, in inches? Express your answer in simplest radical form.



33. ________ (skates) The Skate Sports Store offers 4-wheel and 5-wheel inline skates. On display in the store are the left skates for 17 different styles of skates. If there are 74 wheels in the display, how many of the displayed skates have 5 wheels?

34. ____________ An odd integer between 600 and 800 is divisible by both 9 and 11. What is the sum of its digits?

35. ____________ What is the value of $1 + 7^2 - 7 \cdot 2$?

36. ____________ (black beads) There are 200 red beads and some black beads in a bag. One bead is to be chosen at random. If the probability of selecting a black bead is $\frac{11}{36}$, how many black beads are in the bag?

37. ____________ Mr. Masterson rolls two standard, six-sided dice. What is the probability that he gets a square number on one of the dice and an odd number on the other? Express your answer as a common fraction.

38. ____________ The second and fourth digits of a five-digit integer N are interchanged to form the integer K. What is the remainder when $|N - K|$ is divided by 11?

39. ____________ What is the closest integer to −2.58?

40. ____________ (percent) If the length of a rectangle is increased by 30% and its width is decreased by 20%, by what percent is the area increased?

41. ____________ A rectangle with area of 72 square units has vertices (2, 3), $(2n + 2, 3)$, $(2n + 2, n + 3)$ and $(2, n + 3)$ with $n > 0$. What is the value of n?

42. ____________ (ways) In how many ways can 45¢ be made using any combination of quarters, dimes, and nickels?

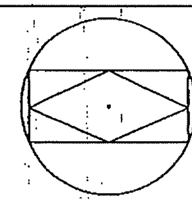
43. ____________ What expression must be in the center cell of the table shown so that the sums of each row, each column, and each diagonal are equivalent?

$2x$	$16x$	$-6x$
$-4x$		$12x$
$14x$	$-8x$	$6x$

44. ____________ (percent) On a true-false test, the ratio of true answers to false answers is 5:3. If Ethan answers all of the questions as "true," what percent of his answers will be correct? Express your answer as a percent to the nearest tenth.

45. ____________ (nickels) John has a total of $1.40 in nickels and dimes, with twice as many nickels as there are dimes. How many nickels does John have?

46. ______ (units) A rectangle with a length and width of 8 units and 6 units, respectively, is inscribed in a circle with diameter 10 units. A rhombus is inscribed in the rectangle. The rhombus is formed by connecting the midpoints of the sides of the rectangle, as shown. What is the perimeter of the rhombus, in units?

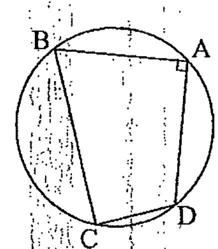


47. ______ If $y = \frac{3}{4}x + 1$, what is the value of x when $y = -3$? Express your answer as a common fraction.

48. ______ In a list of 18 numbers, four of the numbers are increased by 4, and four of the numbers are increased by 5. By how much is the mean increased?

49. ______ What is the largest positive integer that is a factor of every 4-digit even palindrome?

50. ______ (cm) Quadrilateral ABCD is inscribed in a circle, as shown. If $m\angle A = 90°$, CD = 10 cm and BC = 24 cm, what is the radius of the circle, in centimeters?



51. ______ (percent) In a class of 20 students, 11 are girls. What percent of the students are boys?

52. ______ If the least common multiple of a and b is 20, what is the least common multiple of $15a$ and $15b$?

53. ______ (units) A right triangle has two sides of lengths 5 units and 12 units. What is the number of units in the least possible length of the third side? Express your answer to the nearest whole number.

54. ______ A certain number n is tripled and then increased by five. The result is doubled and decreased by 4. If the final result is 36, what is the value of n?

55. ______ What is the product of 18 and $0.\overline{1}$?

56. ______ Given that $48 \le n \le 162$ and $24 \le d \le 36$, what is the product of the smallest and largest possible values for the fraction $\frac{n}{d}$?

57. ______ (percent) Before being sold, a \$75.00 video game was discounted twice by the same percent. The final reduced price was \$27.00. By what percent was the price reduced each time?

58. ______ (cm^2) In the figure shown, adjacent sides of the parallelogram are 4 cm and 6 cm. The indicated angle is 30°. What is the number of square centimeters in the area of the parallelogram?

59. ______ What is the largest integer that is a solution of $-4x > 20$?

60. ______ (degrees) In an isosceles trapezoid, one angle is $\frac{2}{3}$ the size of another angle. What is the number of degrees in the smaller of the two angles?

61. ______ (factors) How many distinct positive integer factors does 12 have?

62. ______ What is the probability that flipping two coins will result in two tails? Express your answer as a common fraction.

63. ______ (minutes) Sabina ran a 10-kilometer race at an average speed of 12 kilometers per hour. How many minutes did it take her to complete the race?

64. ______ When a number n is increased by 50% and the resulting number is divided by 3, the result is 19. What is the value of n?

65. ______ If $3x + 7 = 10 - 2y$, what is the value of $6x + 4y$?

66. ______ When a positive integer is divided by 18, the remainder is 8. When the quotient is expressed as a decimal, what digit is in the ten-thousandths place?

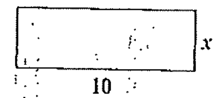
67. ______ The ratio of the length to width of a given rectangle is 3:2. If the length of this rectangle is doubled, what is the ratio of the area of the original rectangle to the new rectangle? Express your answer as a common fraction.

68. ______ What is the sum of the distinct factors of the largest 2-digit square integer?

69. ________ (sides) How many sides does a polygon have if the sum of the measures of the interior angles is 1080 degrees?

70. ________ What is the product of x^2 and the reciprocal of x? Express your answer in terms of x.

71. ________ If the area and perimeter of this rectangle are numerically equal, what is the value of x? Express your answer as a decimal to the nearest tenth.



72. ________ The first sequence shown here is arithmetic, and the second sequence is geometric. After 12 what is the next term the sequences have in common?

3, 12, 21, ...
3, 6, 12, ...

73. ________ Joshua chose a two digit positive integer. He added 10 to the number, and then multiplied the sum by 11. The result was less than 1000. What is the largest possible value of Joshua's result?

74. ________ If $8^2 = 2^x$, what is the value of x?

75. ________ What common fraction is equivalent to $0.\overline{54}$?

76. ________ What is the smallest positive integer value of n for which $(0.5)^n$, when simplified, will have a zero immediately to the right of the decimal point?

77. ________ (pm) A movie that starts at 2:45 pm is 2 hours, 22 minutes long. What time will the movie end?

78. ________ (in^3) The length, width and height of a rectangular prism are in the ratio 3:2:1. The length is 9 in. In cubic inches, what is the volume of the prism?

79. ________ What is the product of all positive integer factors of 10?

80. ________ (m^2) The length of a rectangular playing field is three times the width. If the perimeter of the playing field is 96 m, what is the area of the field in square meters?

Sprint Round Answers

1. 6 cm^2	**11.** $\frac{11}{36}$	**21.** $16\frac{1}{2}$ $units^2$
2. 225	**12.** −5	**22.** 24 units
3. 8	**13.** 6 sides	**23.** π in
4. 1.1 km	**14.** 2	**24.** $\frac{15}{14}$
5. 6 orders	**15.** 212 brownies	**25.** 18π cm^3
6. 3 minutes	**16.** (−2, 0)	**26.** 37.5%
7. 65	**17.** 4	**27.** 4
8. 33	**18.** $\frac{729}{1000}$	**28.** 5
9. 89	**19.** 71 in	**29.** $\frac{1}{3}$
10. 4 pairs	**20.** $\frac{8}{9}$	**30.** 4 units

Target Round Answers

1. 10 combinations	**3.** 36 rectangles	**5.** 12 units	**7.** −62
2. 93	**4.** 10 cups	**6.** 36 degrees	**8.** 31.1 in^2

Team Round Answers

1. 25 cm	**6.** 36 pkgs
2. 125	**7.** 11
3. 4	**8.** 15 points
4. 4%	**9.** (2, 5)
5. 56 cm^2	**10.** 0

Countdown Round Answers

1. $\frac{20}{9}$	16. $\frac{3}{2}$	31. $\frac{1}{729}$	46. 1650 (ft^2)
2. 0	17. 102 (degrees)	32. 4	47. 6 (hours)
3. 77	18. 0	33. 50 (points)	48. 62
4. 180 (minutes)	19. 40 (%)	34. 29 (people)	49. 65 (in)
5. 18 (degrees)	20. 85,000.00 or 85,000 (dollars)	35. 1298	50. 18 (edges)
6. 70 (%)	21. 17 (paper clips)	36. 225	51. 4
7. 6 (integers)	22. 20 (%)	37. 8 (cm)	52. 13
8. 40 (%)	23. 16 (units)	38. 15 (ways)	53. 60
9. $\frac{1}{6}$	24. 125	39. 5.05 (dollars)	54. 6 (ways)
10. c	25. 16 (mi/h)	40. 11	55. $-\frac{1}{2}$
11. 18.8 (minutes)	26. 120 (yd)	41. 48,000.00 or 48,000 (dollars)	56. 48
12. $\frac{1}{2}$	27. 17 (rubber bands)	42. 9 (in)	57. 27
13. 90 (degrees)	28. $\frac{11}{32}$	43. 10 (combinations)	58. 45
14. $\frac{23}{81}$	29. $\frac{3}{8}$	44. 17 (units)	59. 6 (tiles)
15. −8	30. 4 (stools)	45. 0	60. 28 (chords)

