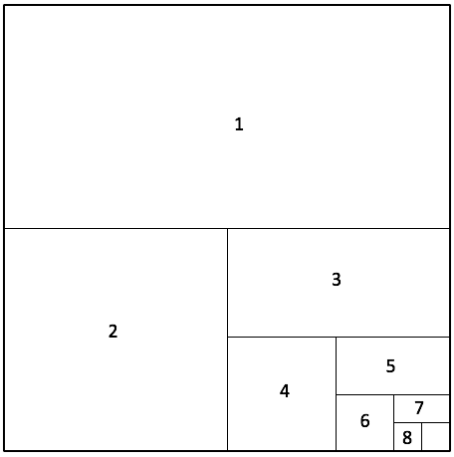
Mathletes Problem of the Week #16

Area Fraction Fractal



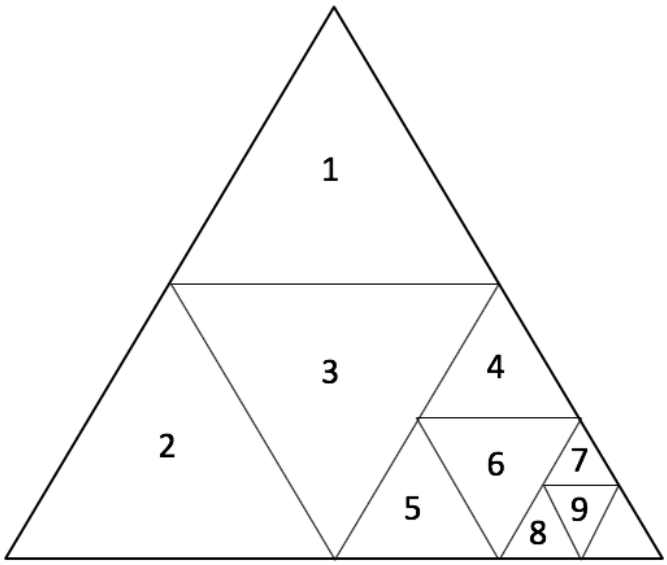
Imagine the large square above is divided into two equal-sized rectangles and then the bottom rectangle is divided into two equal-sized squares. Next, the bottom right square is divided up in the same way the original square was. Now, imagine this continues over and over infinitely. This creates a **fractal**-type structure, where each new section of the bottom right corner is a reduced-size copy of the whole original shape. Let's explore the relative sizes of these smaller areas in the shape.

a) We can think of the sizes of the smaller sections as fractions of the original shape. What fraction of the original shape is each of the numbered sections? What pattern do you see? What expression could you use to find what fraction of the whole shape the n^{th} section represents?
b) Assume the original square has side lengths of 10 units. What is its area? What is the area of the other numbered sections? Do you see the same pattern that you found above? Can you write an expression to find the area of the n^{th} section?
c) What is the sum of sections 1 and 2 together? (Try calculating this using the fractional values you found earlier.) What is the sum of sections 1, 2 and 3? How about sections 1, 2, 3 and 4? Sections 1, 2, 3, 4 and 5? What is the pattern here? Can you explain it in terms of the picture? Can you write an expression for the sum of sections 1 through n?
d) How many of the sections would we have to add together to get more than 99% of the original square's area?
e) Try exploring some of the same questions with the new shape on the back!

Solutions & Explanations: (Try one or try them all! Record your answers below, on the back or a separate sheet of paper.)

Name _____ **Class** _____

(First and last name, please!) **Solutions due: April 17th**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